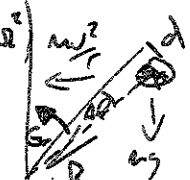




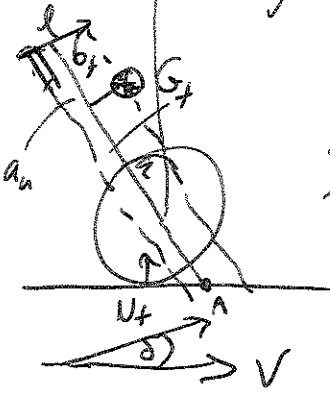
EXAMINATION SOLUTIONS: SESSION 2014/15

Programme:	BEng Motorcycle Engineering (M)	Year:	Level 6
Module/Unit:	Advanced Motorcycle Dynamics		
Author:	Dr Owen Williams	Page:	of

Question No	Question	Marks
1a	 $\tan \theta = \frac{mv^2}{mrg} \quad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ $\theta = 60.56^\circ$	2
b	$G_m = I \omega^2 \cos \theta = (0.48 + 0.58) \times 0.315 \times \frac{36.11}{75} \times \frac{36.11}{75} \times \cos 60.56$ $= 1.06 \times 114.64 \times 0.48 \times \cos 60.56$ $G_m = 28.76 \text{ Nm}$	4
c	 $\text{Torque} = F r d = d \times \sqrt{(m r \omega^2)^2 + (mg)^2}$ $\text{Torque} = G_m = I \omega^2 \cos \theta$ $d = \frac{I \omega^2 \cos \theta}{\sqrt{(mg)^2 + (m r \omega^2)^2}}$	6
d	$d = \frac{1.06 \times 114.64 \times 0.48 \times \cos 60}{\sqrt{(215g)^2 + (215 \times 75 \times 0.48)^2}} = 6.83 \text{ mm}$	4
e	$\Delta \theta = \sin^{-1}\left(\frac{d}{r}\right) = \sin^{-1}\left(\frac{0.00683}{0.65}\right) = 0.6^\circ$	4

Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	of
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Question No	Question	Marks
2a	The capsize motion is in essence a roll motion influenced by the rider action on the bars. The motion is always unstable. Similar to an upturned pendulum, modelled as $\phi = \phi_0 e^{\epsilon/\tau}$ where τ is the positive time constant.	
b	Wobble is the oscillation of the front assembly about the steer axis that can become unstable at fairly low to mid speeds.	
c	Weave is an oscillation of the whole motorcycle, but mainly an effect at the rear of the bike, depending on CG, wheel masses, ground trail (rear and cornering stiffness).	⑥
d		③
e	 $I_A \ddot{\phi} = -C\dot{\phi} - F_s a_n + M_g b_f \delta \sin \epsilon + N_t a_n \delta \sin \epsilon$	④
f	$100^\circ/\text{sec} = \frac{5}{9} \pi \cdot 100/\text{s}$ $\omega = \frac{V}{r} = \frac{55}{0.31} = 177.42$ $M_g = I \omega^2 \cos \phi = 0.42 \times 177.42^2 \times \frac{5}{9} \pi \times \cos 12$ $= 127 \text{ Nm}$	⑦

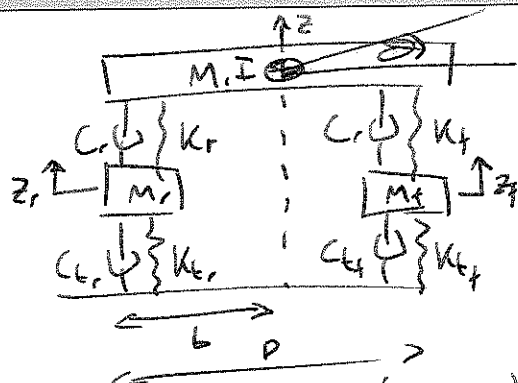
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Module/Unit:	Advanced Motorcycle Dynamics	Page:	of
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Question No	Question	Marks
2b	It can be seen that there is very little change in Steer angle at any point of the lap apart from the slow 'flip-flop' ~~steer~~ ~~duane~~ ~~duane~~. This is consistent with our understanding of dynamics as the steer torque maps instantly to the roll axis at high speed so the steer angle is minimal on the data trace. Steer angle is only seen at low speed.	⑥
C	The roll and steer rates are measured by gyroscopes mounted on the steering head and on the Col G measuring about the roll axis. The direct connection between steer and roll can be seen in those two channels and the direct correlation is evident. It is clear from this that the gyroscopic principle of motorcycle steering is correct and that the instant interrelation between the ~~steer~~ ~~steer~~ ~~steer~~ is clearly seen in the data traces.	⑥
		20

Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	of
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Question No	Question	Marks
5a	 Resolving forces (unchanged) $M\ddot{z} + (K_f + K_r)z + (p-b)K_e\theta - bK_e\theta - K_f z_e - K_r z_r = 0$ $\Rightarrow M\ddot{z} + (K_f + K_r)z + ((p-b)K_e - bK_e)\theta - K_f z_e - K_r z_r = 0$ Rotationally: $I\ddot{\theta} + ((p-b)K_e - bK_e)z + ((p-b)^2 K_e + b^2 K_r)\theta - (p-b)K_e z_e + bK_e z_r = 0$ For M_e $M_e \ddot{z}_e - K_e z - (p-b)K_e \theta + (K_f + K_{e_f})z_e = 0$ For M_r $M_r \ddot{z}_r - K_r z + bK_e \theta + (K_r + K_{e_r})z_r = 0$ $\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & M_e & 0 \\ 0 & 0 & 0 & M_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_e \\ \ddot{z}_r \end{bmatrix} + \begin{bmatrix} K_f + K_r & (p-b)K_e - bK_e & -K_f & -K_r \\ (p-b)K_e - bK_e & (p-b)^2 K_e + b^2 K_r & -(p-b)K_e & bK_e \\ -K_f & -(p-b)K_e & K_f + K_{e_f} & 0 \\ -K_r & bK_e & 0 & K_r + K_{e_r} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ z_e \\ z_r \end{bmatrix} = 0$ including C is simply a repeat of K, just replacing K with C for each case to get $M\ddot{z} + C\dot{z} + Kz = 0$	(4) (6) (8)

Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	of
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Question No	Question	Marks
4a	Equation reduces to $r^2 + 2\zeta\omega r + \omega^2 = 0$ ($x = e^{rt}$) quadratic $\Rightarrow r = \omega(-\zeta \pm \sqrt{\zeta^2 - 1})$ $\zeta < 1$ So $\zeta^2 - 1$ is negative, so complex roots. $r = \omega(-\zeta \pm i\sqrt{1-\zeta^2})$ ($\beta = \sqrt{1-\zeta^2}$) so $r = -\omega\zeta \pm i\omega\beta$ Now since $r = \alpha + i\beta \Rightarrow x = e^{\alpha t} [A\cos(\beta t) + B\sin(\beta t)]$ So $x = e^{-\zeta\omega t} [A\cos(\beta t) + B\sin(\beta t)]$ when $t=0, x=0$ so $0 = e^0 [A\cos 0 + B\sin 0]$ $A = 0$ so $x = B e^{-\zeta\omega t} \sin(\beta t)$ Now $\dot{x} = -\zeta\omega B e^{-\zeta\omega t} \sin(\beta t) + B\beta\omega e^{-\zeta\omega t} \cos(\beta t)$ Now when $t=0$ $\dot{x} = \beta\omega$ $\beta\omega = 0 + B\beta\omega$ so $B = 1$ Now $A=0, B=1$ $x = e^{-\zeta\omega t} [A\cos(\beta t) + B\sin(\beta t)]$ <u>$x = e^{-\zeta\omega t} \sin(\beta t)$</u>	(8)

Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	of
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Question No	Question	Marks
4b	The main points to note are that for the majority of the lap, the steer angle shows only minor variation as the motorcycle progresses around the circuit. It can be seen that the speeds are quite high so a reasonable lean angle would be required, but apart from the one slow corner on the circuit, there is very little in terms of steer angle used. This supports the theory that it is the torques applied at the bars, effecting the lean of the front wheel as a gyroscope, is the dominant factor in the steering of the machine.	6
4c	The data shows both the steer rate and the roll rate as measured by the gyro sensors deployed on the motorcycle. These sensors are measuring the 'rate' of steer and roll in degrees/second. What can be seen here is the direct relationship between action at the steering head and the corresponding roll rate of the motorcycle. This reinforces the point made in 4b that the steering acts by means of torque applied to a gyroscope, but also shows the direct relationship and instantaneous transfer about the gyroscope axes due to the conservation of angular momentum in a gyroscope system	6
		<u>Total 20</u>